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(54) Title: METHOD FOR THE EXPONENTIATION OR SCALAR MULTIPLICATION OF ELEMENTS

(57) Abstract: In order to further develop a method for the multi-exponentiation ( $\prod_{i=1}^d g_i^{e_i}$ ) or the multi-scalar multiplication ( $\sum_{i=1}^d e_i g_i$ ) of elements ( $g_j$ ) by means of in each case at least one exponent or scalar ( $e_i$ ), in particular an integer exponent or scalar, which has in each case a maximum bit rate ( $n$ ) or bit length, in particular for the exponentiation ( $g^e$ ) or scalar multiplication ( $e \cdot g$ ) of an element ( $g$ ) by means of at least one exponent or scalar ( $e$ ), in particular an integer exponent or scalar, which has in each case a maximum bit rate ( $n$ ) or bit length, which elements ( $g_i$ ;  $g$ ) derive from at least one group ( $G$ ), for example an Abelian group, which - in the case of (multi-)exponentiation is notated in particular multiplicatively and - in the case of (multi-)scalar multiplication is notated in particular additively, in such a way that the requirement in terms of storage space for recoded exponents or scalars ( $e_i$ ) is reduced as much as possible even and especially in extremely restricted environments, such as in smart cards for example, the following method steps are proposed: [a.1] computing and storing or [a.2] retrieving from at least one memory all powers ( $g_i^c$ ) or all multiples ( $c \cdot g_i$ ), wherein  $c$  is a permissible positive coefficient; [b] dividing each exponent or scalar ( $e_i$ ) into a number of chunks or into a number of parts ( $e_{i,k}$ ) having a chunk or part width defined by a specific bit rate ( $L$ ); and [c] individually recoding the chunks or parts ( $e_{i,k}$ ).



WO 2005/088440 A1